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FLOW AND HEAT TRANSFER IN THE THERMOGRAVITATIONAL GENERATION MODE

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Experimental investigations [1-3] of heat elimination in a turbulent descending fluid flow in vertical heated pipes under conditions of substantial influence of thermogravitation show that the Nusselt number grows monotonically with the increase in the Grashof number for a constant value of the Reynolds number. The analysis performed in [4] for the case of relatively weak influence of thermogravitation showed that the monotonic increase in the Nusselt number in this case is associated with the influence of lift forces on the turbulent transfer under conditions of unstable stratification of the density. The influence of thermogravitation results in an increase in turbulent transfer, in a more filled-out shape of the velocity and temperature profiles, in an increase in the friction drag and heat elimination. The nature of the flow is hence determined for all values of the Grashof number by the influence of the thermogravitational forces on the turbulent transfer. The influence of the thermogravitational forces directly on the average flow (i.e., taking account of the lift forces in the average equation of motion) is substantially less than their influence on turbulence for relatively low values of the Grashof number, and this difference increases more and more with its growth.

The expression in [4] for the coefficient of turbulent momentum transfer ε,

$$\frac{\varepsilon}{\nu}=\left(\frac{\varepsilon}{\nu}\right)_{\mathrm{T}}\left[1+41\frac{\mathrm{Gr}}{\mathrm{Re}_*^4\mathrm{Pr}}\,\frac{dT^+/d\eta}{\mathrm{Pr}_{\mathrm{T}}(dU^+/d\eta)^2}\right]^{1/4},\tag{1}$$

was used for the case of smallness of the parameter taking account of the lift forces, i.e., when the second member in the square brackets is substantially less than one. Here ν is the kinematic coefficient of viscosity; $(\varepsilon/\nu)_T$ is the relative coefficient of turbulent momentum transfer in an isothermal flow; $\mathrm{Gr} = g\beta q_w d^4/\lambda\nu^2$ is the Grashof number; $\mathrm{Re}_* = v_* d/\nu = \mathrm{Re}\sqrt{c_f/2}$, $\mathrm{Re} = \bar{u}d/\nu$ is the Reynolds number; β is the coefficient of volume expansion; g is the acceleration of gravity; q_w is the thermal-flux density at the wall; d is the pipe diameter (characteristic dimension); λ is the coefficient of thermal conductivity; $v_* = \sqrt{\tau_w/\rho}$ is the dynamic velocity; τ_w is the tangential friction stress at the wall; ρ is the density; cf is the friction drag coefficient; $\bar{u}$ is the mean (characteristic) velocity with respect to the transverse section; Pr is the Prandtl number; Pr_T is the turbulent Prandtl number; $T^+ = (t_w - t)\cdot\rho c_p v_*/q_w$ is the dimensionless temperature; t is the temperature; t_w is the wall temperature; c_p is the specific heat at constant pressure; $\eta = v_* y/\nu$ is the dimensionless distance to the wall; y is the distance from the wall along the normal; $U^+ = u/v_*$ is the dimensionless velocity; and u is the velocity parallel to the wall in the x direction.

The second member in the square brackets in (1) characterizes the contribution of thermogravitation to the generation of turbulence as compared with generation because of the average flow, and this member is considered substantially greater than the first term in this paper. This corresponds to the following physical situation when generation of turbulence by the thermogravitational forces is substantially greater than generation by the average flow. We call such a mode the "thermogravitational generation mode." The forced average flow is hence accomplished because of an external circulation source.

Equation (1) is written for the thermogravitational generation mode as

$$\frac{\varepsilon}{\nu}=\frac{C}{\mathrm{Pr}_{\mathrm{T}}^{1/4}}\,\frac{\mathrm{Gr}^{1/4}}{\mathrm{Re}_*\mathrm{Pr}^{1/4}}\,\frac{(dT^+/d\eta)^{1/4}}{(dU^+/d\eta)^{1/2}}\left(\frac{\varepsilon}{\nu}\right)_{\mathrm{T}},\tag{2}$$

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where C is a constant [the constant is C = 2.6 in conformity with the value of the numerical coefficient in (1), however, its value requires refinement because of comparison of the solution with test data].

By using (2) we obtain the following expressions for the tangential friction stress and the heat flux density:

$$\frac{\tau}{\tau_w}=\left(1+\frac{\varepsilon}{\nu}\right)\frac{dU^+}{d\eta}=\left[1+\frac{C}{\mathrm{Pr}^{1/4}}\frac{\mathrm{Gr}^{1/4}}{\mathrm{Re}_*\mathrm{Pr}^{1/4}}\frac{(dT^+/d\eta)^{1/4}}{(dU^+/d\eta)^{1/2}}\left(\frac{\varepsilon}{\nu}\right)_\mathrm{T}\right]\frac{dU^+}{d\eta}; \tag{3}$$

$$\frac{q}{q_w}=\left(\frac{1}{\mathrm{Pr}}+\frac{1}{\mathrm{Pr_T}}\frac{\varepsilon}{\nu}\right)\frac{dT^+}{d\eta}=\left[\frac{1}{\mathrm{Pr}}+\frac{C}{\mathrm{Pr}_\mathrm{T}^{5/4}}\frac{\mathrm{Gr}^{1/4}}{\mathrm{Re}_*\mathrm{Pr}^{1/4}}\frac{(dT^+/d\eta)^{1/4}}{(dU^+/d\eta)^{1/2}}\left(\frac{\varepsilon}{\nu}\right)_\mathrm{T}\right]\frac{dT^+}{d\eta}. \tag{4}$$

Let us consider a two-layer flow model by assuming that the influence of molecular viscosity and molecular thermal conductivity is negligible for values of $\eta > \eta_1$ while turbulent momentum and heat transfer are negligible for $\eta < \eta_1$. The problem is solved under the following assumptions: 1) The tangential friction stress and heat flux density equal, respectively, the wall values i.e., $\tau/\tau_W = 1$ and $q/q_W = 1$; 2) since the structure of the near-wall flow being considered is due mainly to the thermal effect on the stream, we shall define the viscous sublayer as the fluid layer which is stable to thermogravitational perturbations; 3) the thickness of the molecular viscosity layer equals the thickness of the molecular thermal conductivity layer; 4) the coefficient of turbulent momentum transfer $(\varepsilon/\nu)_T$ in the absence of the influence of mass forces is described by the dependence $(\varepsilon/\nu)_T = 0.4\eta$.

From (3) and (4) we find for the velocity and temperature distributions at $\eta \le \eta_1$

$$U^+ = \eta; \tag{5}$$

$$T^+ = \mathrm{Pr}\eta; \tag{6}$$

and for $\eta_1 \le \eta \le \eta_0$

$$U^+ = \eta_1+\frac{C_1}{\gamma^{1/3}}\left(\frac{1}{\eta_1^{1/3}}-\frac{1}{\eta^{1/3}}\right); \tag{7}$$

$$T^+ = \eta_1\mathrm{Pr}+\frac{C_2}{\gamma^{1/3}}\left(\frac{1}{\eta_1^{1/3}}-\frac{1}{\eta^{1/3}}\right), \tag{8}$$

where $C_1 = (10/C^{4/3})\mathrm{Pr}_T^{1/3}$; $C_2 = (10/C^{4/3})\mathrm{Pr}_T^{4/3}$ are constants (a constant value of Pr_T over the stream section is assumed here;

$$\gamma=\frac{\mathrm{Gr}}{\mathrm{PrRe}_*^4}=\frac{\mathrm{Gr}}{\mathrm{PrRe}^4}\left(\frac{2}{c_f}\right)^2=E\left(\frac{2}{c_f}\right)^2;\ \eta_0=v_*d/2\nu.$$

The constants of integration in (7) and (8) are determined from the condition that the velocity and temperature values are equal at the point η_1.

In the majority of papers devoted to investigations in the atmosphere under conditions of strong unstable density stratification, the value $\mathrm{Pr}_T = 0.77$ is obtained (see [5], for example). We shall use this value of Pr_T in the subsequent calculations and, as has been noted above, we take the value of the constant C equal to 2.6, and we hence have $C_1 = 2.8$ and $C_2 = 2.2$. The value of the constant C_2 determined by temperature profile measurements in [1, 6] is close to that mentioned.

Still another unknown parameter η_1 enters into the dependences (5)-(8) in addition to the constants C_1 and C_2 (or C and Pr_T). In the case of isothermal near-wall flow, the thickness of the viscous layer is $\eta_1 = 11.6$ according to the Prandtl two-layer model. We estimate the quantity η_1 for the thermogravitational generation mode from consideration on the convective stability of the near-wall layer.

It has been found in [7] that the Grashof number $\mathrm{Gr}_1^0 = [g\beta(t_W - t_0)\delta_1^3]/\nu^2$, computed over the thickness of the viscous sublayer δ_1 for turbulent free-convective flow along a vertical plate, is independent of the longitudinal coordinate and the temperature difference between the wall (t_W) and the surrounding medium (t_0), but depends on Pr. The following dependence is obtained for Gr_1^0:

$$\mathrm{Gr}_1^0 = 60/\sqrt{\mathrm{Pr}}.$$

Therefore, the combination $\mathrm{Gr}_1^0\sqrt{\mathrm{Pr}}$ can be interpreted as a critical parameter governing the stability of the viscous sublayer in a turbulent free convective stream.

Considering the viscous sublayer as the near-wall layer, which is stable to thermogravitational perturbations, for a strong unstable density stratification we represent the condition for its stability in conformity with the results obtained in [7] as

$$\mathrm{Gr}_1\mathrm{Pr}^n = \frac{g\beta q_w \delta_1^4}{\lambda\nu^2}\mathrm{Pr}^n = B, \tag{9}$$

where B and n are constants.

Expressing δ_1 in terms of the dimensionless coordinate of the viscous sublayer thickness $\eta_1 = (v_*/\nu)\delta_1$, we obtain from (9)

$$\eta_1 = \frac{B^{1/4}}{E^{1/4}\mathrm{Pr}^{(1+n)/4}}\sqrt{\frac{c_f}{2}}. \tag{10}$$

Integrating the expression describing U^+ over the pipe transverse section or taking the value of U^+ outside the boundary layer, we find the dependence for the friction drag coefficient c_f governing the friction drag in these two cases as follows:

$$\Delta P = \frac{4l}{d}\tau_w = \xi\frac{l}{d}\frac{\rho\bar{u}^2}{2}, \quad \xi = 4c_f, \quad \tau_w = c_f\frac{\rho u_0^2}{2}$$

Using (5), (7), and (10), we obtain a dependence for the friction drag coefficient for the boundary layer,

$$\frac{c_f}{2} = E^{1/4}\left[\sqrt{b} + C_1\left(b^{-1/6} - a_\delta^{-1}\right)\right]^{-1}, \tag{11}$$

where

$$b = \sqrt{B/\mathrm{Pr}^{1+n}}; \quad a_\delta = E^{1/12}\mathrm{Re}_\delta^{1/3} = (\mathrm{Gr}_\delta/\mathrm{Pr})^{1/12}; \quad \mathrm{Re}_\delta = \frac{u_0\delta}{\nu};$$

u_0 is the value of the velocity outside the boundary layer, δ is the boundary-layer thickness, $\mathrm{Gr}_\delta = g\beta\delta^4 q_w/\lambda\nu^2$, and for a circular pipe

$$\frac{\xi}{8} = E^{1/4}\left[(1 - Y_1)^2\left(\sqrt{b} + C_1 b^{-1/6}\right) - \frac{2.3C_1}{a}\left(1 - 1.7Y_1^{2/3}\right)\right]^{-1}, \tag{12}$$

where

$$a = E^{1/12}\mathrm{Re}^{1/3} = (\mathrm{Gr}/\mathrm{Pr})^{1/12}; \; Y_1 = \eta_1/\eta_0 = 2\sqrt{b}/a^3.$$

We determine the heat elimination for a flow in the boundary layer and in a circular pipe from the dependences found for the temperature and velocity profiles and the friction drag coefficient. The following expression is obtained for the Stanton number for the boundary layer:

$$\mathrm{St} = q_w/\rho c_p u_0 (t_w - t_0) = E^{1/4}\left[\mathrm{Pr}\sqrt{b} + C_2\left(b^{-1/6} - a_\delta^{-1}\right)\right]^{-1}.$$

The expression for St in a circular pipe is awkward, hence we shall not write it in general form. It simplifies substantially for large values of a:

$$\mathrm{St} = q_w/\rho c_p \bar{u}(t_w - t_b) = \frac{E^{1/4}\left(\sqrt{b} + C_1 b^{-1/6}\right)}{\mathrm{Pr}\, b + C_1 C_2 b^{-1/3} + (C_1 + C_2\,\mathrm{Pr})b^{1/3}},$$

where St = Nu/Pe; Pe = RePr; t_b is the mean mass temperature of the fluid in a given section.

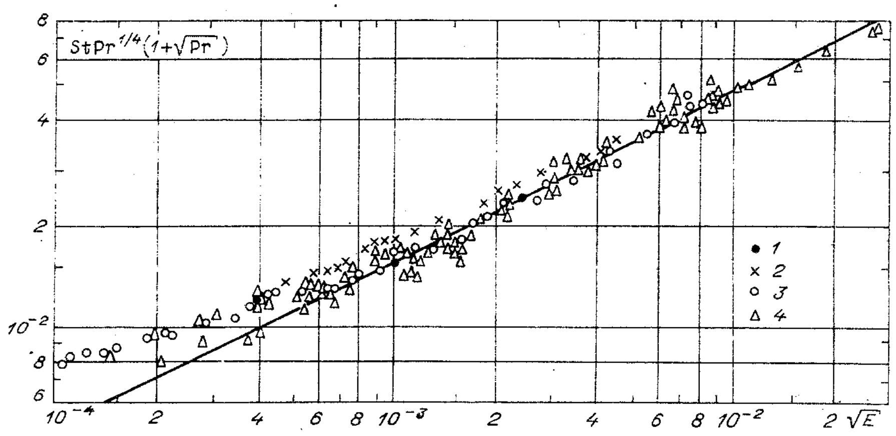

Fig. 1

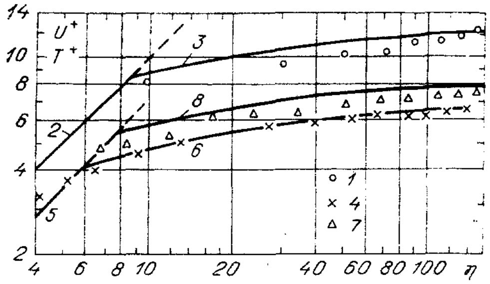

Fig. 2

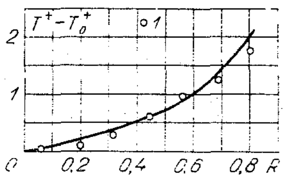

Fig. 3

As a result of generalizing existing test data on local heat elimination during flow from the top down in vertical heated pipes [1-3], and in a boundary layer on a heated plate [6], it is found that B = 120, n = 0.8 for C_1 = 2.8 and C_2 = 2.2.

The computed and experimental results on heat elimination in the thermogravitational generation mode are approximated by the following interpolation dependences in the Pr = 0.5-20 range:

$$\mathrm{St}_g = 0.5E^{1/4}[\mathrm{Pr}^{1/4}(1+\sqrt{\mathrm{Pr}})]^{-1}; \quad (13)$$

$$\mathrm{Nu}_g = 0.5\sqrt{\mathrm{Pr}}\,\mathrm{Gr}^{1/4}[1+\sqrt{\mathrm{Pr}}]^{-1}. \quad (14)$$

The dimensionless viscous sublayer thickness η_1 depends on Pr and the parameter E in the thermogravitational generation mode and, taking account of the data obtained, is described on the basis of (10) by the dependence

$$\eta_1 = [E^{1/8}(1+0.57\,\mathrm{Pr}^{0.6})]^{-1}.$$

The results obtained show that the velocity and temperature profiles, the friction drag, the thickness of the domain of predominant influence of molecular viscosity and thermal conductivity η_1 (or δ_1), the St depend on E and Pr in the case under consideration, while Nu depends on Gr and Pr and is independent of Re.

Given in Fig. 1 is a comparison between test results (points) on local heat elimination for a flow from the top down in vertical heated pipes [1-3] and in the boundary layer on a vertical heated plate [6] and a computation (line) according to the dependence (13). The experimental results [1, 6] shown by the points 1, 2 are obtained for an air flow, and the data [2, 3] shown by the points 3, 4 are obtained for a water flow. The parameter range enclosed by the data presented is Pr = 0.7-8, Re = $(1.2-20)\cdot 10^3$, Gr = $10^6-2\cdot 10^{10}$. The dependence (13) describes the test results satisfactorily for an essential influence of the thermogravitation corresponding to relatively high values of the parameter E. The influence of the lift force diminishes with the diminution in E; the experimental points deviate from the dependence (13) and approach values characteristic of a forced flow without the influence of mass forces.

The velocity and temperature profiles for an air flow from the top down are measured in [1] in a vertical heated circular pipe, and the temperature profiles for a forced air flow from the top down over a vertical heated plate in [6]. A comparison between the experimental results on the velocity and temperature distributions and values computed by the dependences (5)-(8) and (10)-(12) is shown in Fig. 2 in universal coordinates. The test values of the velocity (points 1) obtained in [1] for values of E = 10^{-6} and Re = 2050 are described satisfactorily by the curves 2, 3 computed by means of the dependences (5), (7), and (12). The temperature test results (points 4, 7) obtained in [6] for the values E = $5.4\cdot 10^{-6}$ and $7.4\cdot 10^{-7}$ are, respectively, described satisfactorily by the curves 5-6 and 5-8, computed by means of the dependences (6), (8), and (11).

The velocity and temperature distributions in the exit section of the pipe are measured in [1]. Because of endface heat leakage, the wall temperature in the exit section was reduced substantially. In this connection, the measured temperature profiles cannot be represented in the coordinates $T^+ - \eta$.

Experimental values [1] of the wall temperature (points 1) are compared in Fig. 3 with values computed (lines) by means of the dependences (8), (11) in coordinates measured from the pipe axis for the values E = 10^{-6} and Re = 2050 corresponding to the test results.

Test results on the local heat elimination obtained by different authors in investigations of water flow from the top down (Pr = 3-7) in vertical heated pipes under conditions

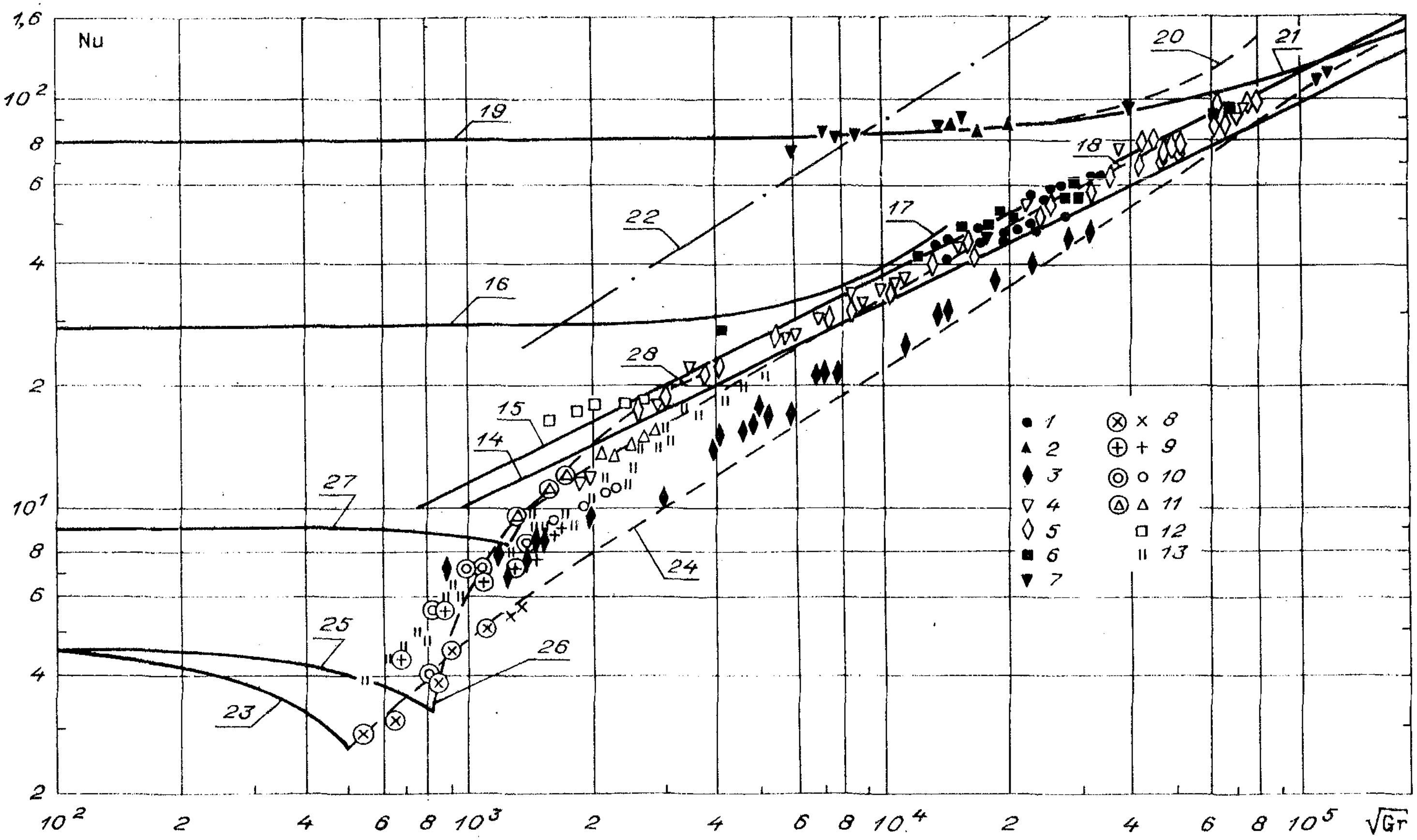

Fig. 4

TABLE 1

Number of symbol in Fig. 4	1	2	3	4	5	6	7	8	9	10	11	12	13
Source	[2]	[2]	[3]	[3]	[3]	[3]	[3]	[8]	[8]	[8]	[8]	[8]	[9]
Pr	5,3—6,8	5,5	2—6	2—6	2—6	2—6	≈5	4—7	4—7	4—7	4—7	4—7	2,8—5,8
$Re \cdot 10^{-3}$	1,1—3,0	≈10	0,3—0,75	0,8—1,5	1,6—2,5	2,6—4,1	10	0,3—0,4	0,6—0,7	0,9—1	1,4—1,7	2,6	<0,9

when disturbance of the laminar flow stability has already occurred are presented in Fig. 4. All the test and computed results presented in Fig. 4 refer to spacings greater than 30 calibers from the beginning of pipe heating.

The influence of the lift forces for forced and free convection acting in the opposite direction at the wall results in disturbance of the laminar flow stability even for small values of R and relative small values of Gr. As a result of the generalization of the available experimental and computed results (particularly the results in [8-11]) and the assumption that the nature of the change in the critical value Gr_* in the initial section is analogous to the nature of the change in heat elimination, the dependence

$$\frac{Gr_*}{Re} = \frac{65\sqrt{2400 - Re}}{[1 - \exp(-40X)][4.36 + 1.31X^{-1/3}\exp(-13\sqrt{X})]} \tag{15}$$

is obtained, which describes the boundary for the disturbance of stability in the range $Re = 10^2$–$2.3 \cdot 10^3$ for any value of the reduced length $X = x/d\ Pe$.

Heat elimination for viscous-gravitational flow to the limit of the stability disturbance (15) is described by the dependence

$$\frac{Nu}{Nu_l} = 1 - 1.3 \cdot 10^{-4}\frac{Gr}{Re} Nu_l [1 - \exp(-40X)] \tag{16}$$

obtained in [11], where $Nu_l = 4.36 + 1.31X^{-1/3}\exp(-13\sqrt{X})$ is the value of the Nusselt number corresponding for any X to the exact solutions with ±3% accuracy for the case of heat transfer in a laminar flow with constant physical properties and a constant heat-flux density at the wall.

The lines 23, 25, and 27 superposed in Fig. 4 have been constructed by means of the dependence (16) for the values Re = 350, X > 0.07; Re = 1550, X > 0.07; Re = 2300, $X = 2.6 \cdot 10^{-3}$ (x/d = 30, Pr = 5).

Results (points 8-12) have been obtained in [8] for the transition from viscous-gravitational to turbulent flow. The first group of points 8-11 (dark circles) refers to modes with intermittent flow while the second group of points 8-12 (without circles) refers to modes with developed fluctuating flow.

Results in [8] have been obtained in a long pipe (around 620 calibers) and corresponds to the case of stabilized flow and heat exchange. However, the Gr values in these tests are insufficient for development of a thermogravitational generation mode for which the dependence of Nu on Gr is shown in Fig. 4 by the lines 14 and 15 constructed by means of (14) for values of Pr equal to 3 and 7, respectively.

The parameters for which the test results represented in Fig. 4 have been obtained are presented in Table 1 (the value of Nu is computed in [9] by means of the difference between the wall temperature and the temperature at the center of the stream).

An isothermal flow is turbulent for numbers $Re > 3 \cdot 10^3$ under conditions of a stream perturbed at the pipe entrance. The influence of the lift forces reduces to intensification of turbulent transfer and growth of the number Nu with the growth in Gr for pipe heating and a flow from the top down (or from the bottom up with cooling). Heat elimination with a small degree of influence of the thermogravitation is described in [4]. The lines 17 and 20 in Fig. 4 have been computed according to the dependences obtained in [4] for the values Pr = 5 and $Re = 3 \cdot 10^3$, 10^4, respectively. Lines 16 and 19 have been computed by means of the dependence obtained in [12] for the values Pr = 5 and $Re = 3 \cdot 10^3$, 10^4, respectively. Line 22 describes the limit of a 1% change in the number Nu for Pr = 5 because of the influence of the lift forces as compared with the number Nu_T for a turbulent flow with constant physical properties.

Because of generalization of the experimental and computed results on heat elimination under the effect of thermogravitation on forced flow from the top down in vertical heated pipes (from the bottom up in cooled pipes), the following interpolation dependences are obtained: for $Re < 2.3 \cdot 10^3$

$$Nu = (1 - 8.5 \cdot 10^{-3}\sqrt{2400 - Re})[4.36 + 1.31X^{-1/3}\exp(-13\sqrt{X}) \times \\ \times \exp\beta + Nu_g[1 - 0.65(Gr_*/Gr)^2](1 - \exp\beta), \tag{17}$$

where $\beta = -6.4 \cdot 10^{-4} Re (Gr/Gr_*)^{1/4}$; Gr_* is computed by means of the dependence (15) and Nu_g by the dependence (14); for $Re > 3 \cdot 10^3$

$$\mathrm{Nu} = \mathrm{Nu_T} \exp(-0.02\mathrm{Gr}/\mathrm{Gr}_b) + \mathrm{Nu}_g[1 - \exp(-0.04\mathrm{Gr}/\mathrm{Gr}_b)], \tag{18}$$

where

$$\mathrm{Gr}_b = 1.3 \cdot 10^{-4}\mathrm{Pr}\mathrm{Re}^{2.75} \frac{\mathrm{Re}^{1/8} + 2.4(\mathrm{Pr}^{2/3} - 1)}{\lg \mathrm{Re} + 1.15 \lg(5\mathrm{Pr} + 1) + 0.5\mathrm{Pr} - 1.8};$$

$$\mathrm{Nu_T} = \frac{\mathrm{Pe}\,\xi/8}{1.07 + 12.7\sqrt{\frac{\xi}{8}}\,(\mathrm{Pr}^{2/3} - 1)}; \quad \xi = 0.316/\mathrm{Re}^{1/4};$$

Nu_g is computed by means of the dependence (14).

The lines 16, 18, and 19, 21 are, respectively, shown in Fig. 4 as computed by the dependences (18) for Pr = 5 and Re = $3 \cdot 10^3$, 10^4, as are the lines 24, 26, and 28 computed by the dependence (17) for the values X > 0.07 and Re = 350, 1550; X = $2.6 \cdot 10^{-3}$ and Re = $2.3 \cdot 10^4$, respectively.

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